9.7 Residual Strength Parametric Analysis

This section illustrates parametric analyses available to an engineer for evaluating the sensitivity of residual strength to geometric and material parameters. As discussed in Section 4, the residual strength relates load carrying capability to material toughness and crack size in a unique way for each structure. Two methods are generally available for describing the residual strength of a structure: the first is with a relationship between residual strength and crack length, and the second is with a relationship between residual strength and time. These relationships are summarized in Figure 9.7.1. The first relationship is best used to describe the effects of toughness or of global geometry. The second relationship is best used to describe the effects of crack growth resistance and of global geometry.

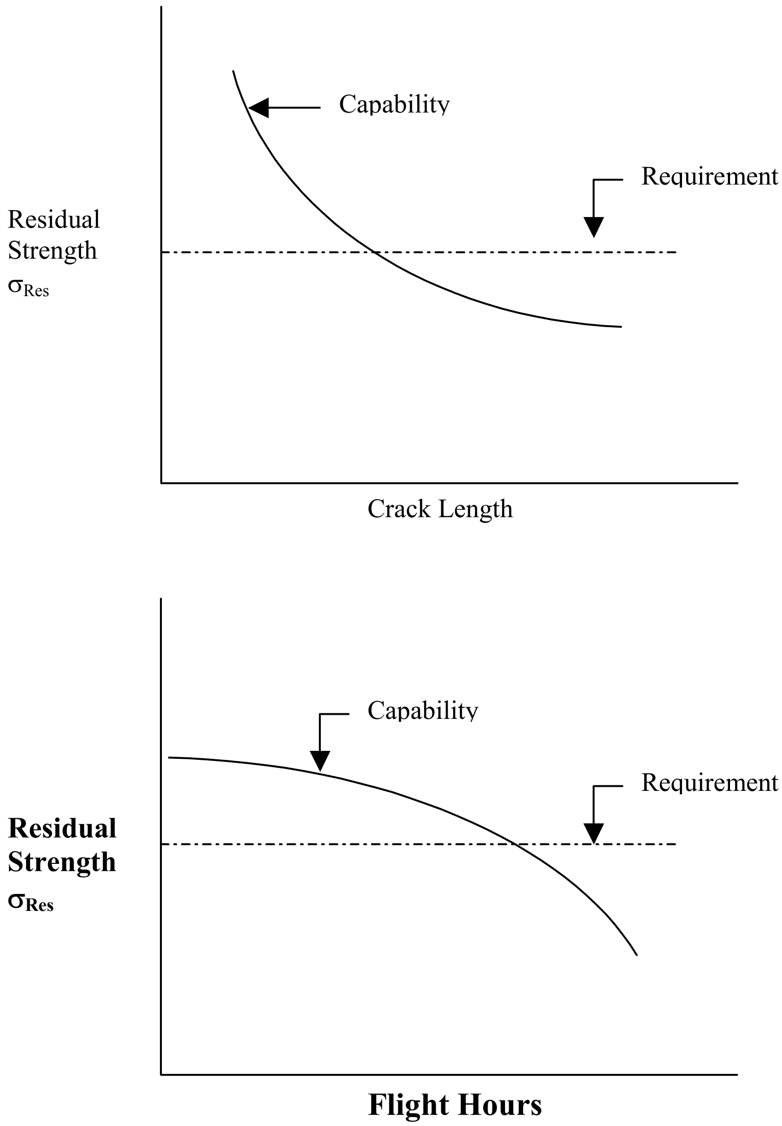


Figure 9.7.1. Types of Residual Strength Relationships

Constructing residual strength-crack length relationships are relatively straight forward. To do so requires both fracture toughness data for the material and a stress-intensity factor analysis for the cracked structure. Fracture toughness data can be found in the Damage Tolerant Design (Data) Handbook [Skinn, et al., 1994]. The stress-intensity factor for a given cracked structure can be obtained through the methods discussed in Section 11.

Constructing residual strength-life relationships requires the same information as above plus a description of the crack growth life behavior under the service loading. This additional information can be generated by integrating wide area crack growth rate equations or by summing incremental damage on a cycle-by-cycle basis. Cycle-by-cycle damage summation presumes (a) that a stress history is available for the cracked structure and (b) that constant amplitude fatigue crack growth rate data are available for the material. The additional complexity associated with generating residual strength-life relationships is one reason why residual strength data are normally only presented as a function of crack length.

A series of examples have been prepared to describe the effects of material properties, spectrum stress level, and structural geometry on the residual strength of relatively simple structures. The approach taken could be duplicated for other more complicated situations related to specific structural repairs.

For each example, the Irwin abrupt fracture criterion is employed to obtain the relationship between residual strength and crack size. Simply stated, failure is presumed to occur when the applied stress-intensity factor (K) is greater than or equal to the fracture toughness (K_{cr}) of the material, i.e.

$$K \geq K_{cr} \tag{9.7.1}$$

then failure occurs. Because the stress-intensity factor is a function of stress and crack size, Equation 9.7.1 provides the relationship between residual strength and critical crack length.

To facilitate a general overview of the residual strength-life relationship, the wide area crack growth rate equation methods developed in Section 9.3 are utilized. In this section, the wide area equation is expressed as

$$\frac{da}{dN} = C\overline{K}^{n} \tag{9.7.2}$$

where the crack growth rate (da/dN) is given appropriately as a function of cycles, flights, or flight hours depending on the given structural situation. Also, the characteristic stress-intensity factor ($\overline{K}$) in Equation 9.7.2 is related to the characteristic stress ($\overline{\sigma}$) through

$$\overline{K} = \overline{\sigma}\left(\frac{K}{\sigma}\right) \tag{9.7.3}$$

where (K/σ) is the stress-intensity factor coefficient (dependent only on geometric parameters, such as crack length and edge distances). The residual strength-life relationship is obtained by cross correlating the residual strength calculated from Equation 9.7.1 with the life calculated from

$$LIFE = \int_{a_o}^{a_{cr}} \frac{da}{C\overline{K}^{n}} \tag{9.7.4}$$

The cross correlation is accomplished using the same value of critical crack length (a_{cr}) for both the residual strength and life calculations.

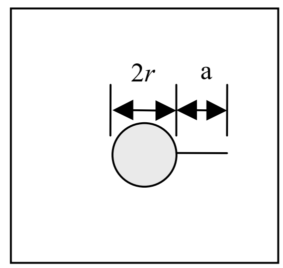
Example 9.7.1 considers the effect of fracture toughness on both the residual strength-crack length and residual strength-life relationships. These relationships are established for an open hole with a through-the-thickness radial crack, and for a wide area crack growth rate equation defined as

$$\frac{da}{dN} = 1 \times 10^{-8} \overline{K}^{3.0} \tag{9.7.5}$$

Examples 9.7.2, 9.7.3, and 9.7.4 consider how the characteristic stress level ($\overline{\sigma}$), the constant C, and the exponential constant n, respectively, affect the residual strength-life relationship. Subsequently, Examples 9.7.5 and 9.7.6 present the effect of geometrical and loading changes on the residual strength-crack length and residual strength-life relationships.

EXAMPLE 9.7.1 Effect of Fracture Toughness

Evaluated the residual strength relationships be as a function of fracture toughness for the structural geometry and loading described in the figure shown here.



$a_0 = 0.050$ inch
$r = 0.125$ inch
$K_{IC} = 30$ ksi $\sqrt{\text{in}}$
$\frac{da}{dN} = 1 \times 10^{-8} \overline{K}^{3.0}$

To determine the residual strength-crack length relationship, Equation 9.7.1 is utilized in conjunction with the stress-intensity factor coefficient obtained from Section 11. The stress-intensity factor coefficient for the tension-loaded, open-hole with a radial-through-the-thickness crack in a wide plate is given by:

$$\frac{K}{\sigma} = \sqrt{\pi a}\left(0.7071 + 0.7548y + 0.3415y^2 + 0.642y^3 + 0.9196y^4\right)$$

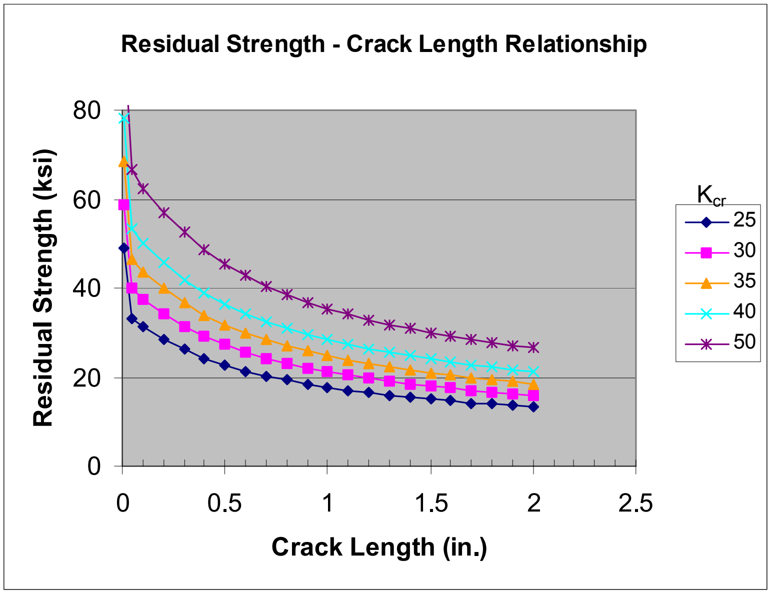
where $y = \left(\frac{1}{1 + a/r}\right)$

Solving Equation 9.7.1 in conjunction with this equation leads to the relationship between residual stress and critical crack size, thus

$$K_{cr} = \sigma_{cr}\sqrt{\pi a_{cr}}\left(0.7071 + 0.7548y + 0.3415y^2 + 0.642y^3 + 0.9196y^4\right)$$

where $y = \left(\frac{1}{1 + a_{cr}/r}\right)$

Defining a series of critical crack sizes for a given value of K_{cr} is the easiest method for evaluating the relationship. The following plot describes the relationship between residual strength and crack length, evaluated in this manner, for several given values of K_{cr}. As the plot illustrates, a substantial difference exists between the residual strength curves at any crack length; this difference is linearly related to the fracture toughness level. For a defined crack length, the equation gives a constant value of (K/σ), so an increase in K_{cr} leads to a similar increase in σ_{res}.



Effect of Fracture Toughness on the Residual Strength-Crack Length Relationship for a Radially Cracked Hole

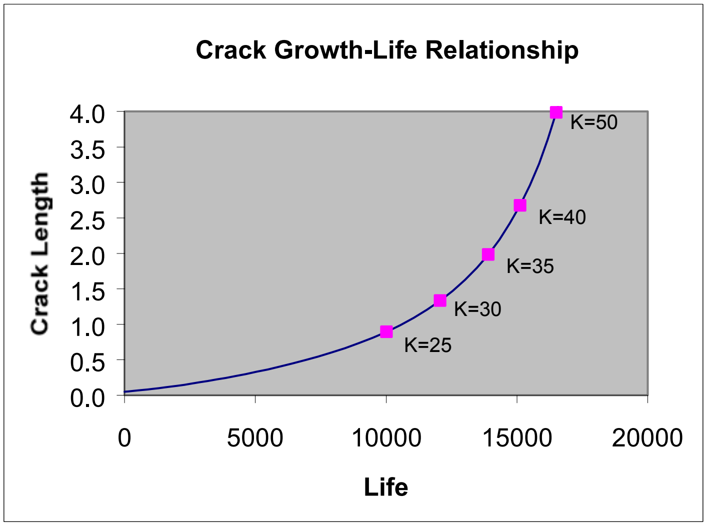
Evaluating the difference between the residual strength curves based on a constant residual strength level illustrates that a greater improvement factor on critical crack size accompanies an increase in fracture toughness. For a requirement of σ_{res} = 40 ksi, the critical crack size for the K_{cr} = 30 ksi $\sqrt{\text{in}}$ material is about 0.050 inch, whereas that for a K_{cr} = 50 ksi $\sqrt{\text{in}}$, the critical crack size is about 0.750 inch. As a first order approximation of the improvement factor, one might neglect the influence of the β factor and arrive at a simplified ratio

$$\frac{a_{cr}^{new}}{a_{cr}^{old}} = \left(\frac{K_{cr}^{new}}{K_{cr}^{old}}\right)^2$$

that illustrates the reason for the dramatic increase in critical crack length for fracture toughness improvements.

One function of an engineer is to provide the structure with sufficient fracture toughness in order to maintain the required residual strength throughout the anticipated service lifetime. A choice of high fracture toughness is appropriate when the engineer is attempting to ensure that potentially damaging cracks are large and easily inspectable prior to the loss of a residual strength requirement. To determine how rapidly the residual strength decays, it is necessary to perform a crack growth life calculation.

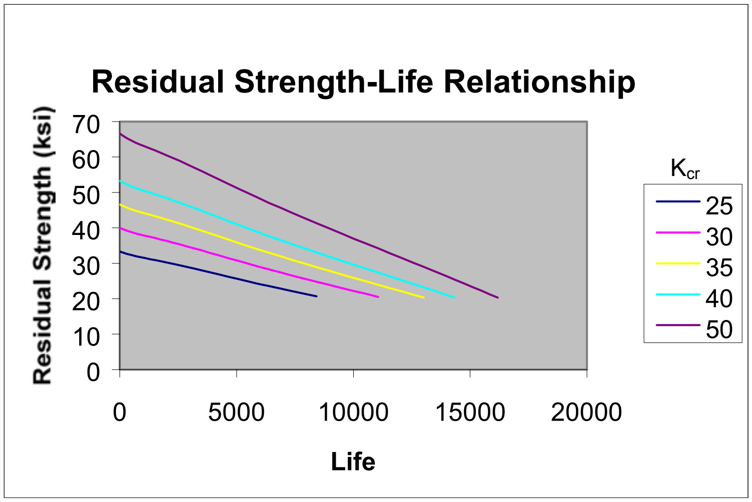
When the crack growth life calculation is based on the integral formulation of Equation 9.7.2 (a power law), i.e. on Equation 9.7.4, the shape of the crack growth-life curve is as shown here. This specific curve was obtained for a characteristic stress level ($\bar{\sigma}$) of 20 ksi and employed $C = 1 \times 10^{-8}$ and $n = 3.0$ as the constants in the growth rate equation. The initial crack length (a_o) was chosen as 0.050 inches.



Crack Growth-Life Relationship for the Baseline Geometry

The curve has been marked to indicate the stress-intensity factor at various crack length levels. These levels correspond to the lower fracture toughness levels shown in the previous plot. One consequence of using a power law equation to describe crack growth rate behavior is that the crack growth life curve does not indicate a rapid increase as the stress-intensity factor approaches the fracture toughness level. From a practical standpoint, only a slight error in the life calculation occurs due to inaccurately modeling the crack growth rate in the fracture toughness regime.

When the crack length-life data are cross correlated with the residual strength-crack length data, one obtains the relationships between residual strength and life shown below. Each residual strength-life data point is associated with a common crack length that relates the data in the previous figures. The figure shows that the highest values of fracture toughness are again associated with the highest values of residual strength. The figure also shows that a material with high fracture toughness will maintain a high residual strength capability longer than one with low fracture toughness, all other conditions being equal. Interestingly, for the conditions given for this example, the residual strength capability decays in a linear fashion for most of the life. The only non-linearity occurs in the earliest part of life where the crack is in a severe stress-intensity factor gradient. Other factors which affect the extent of the nonlinear region will be discussed later.



Effect of Fracture Toughness on the Residual Strength-Life Relationship

EXAMPLE 9.7.2 Effect of Characteristic Stress Level

Because the operational stress level significantly affects the crack length life of a structure, an engineer might wish to consider its effect on residual strength. For this evaluation, assume that the material is known to have a fracture toughness (K_{cr}) of 30 ksi √in and a crack growth rate behavior given by Equation 9.7.5 as:

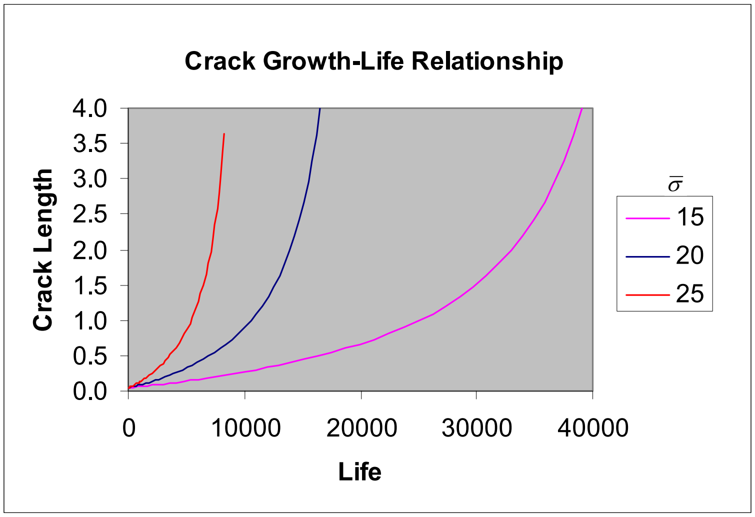
$$\frac{da}{dN} = 1 \times 10^{-8} \overline{K}^{3.0}$$

The residual strength-crack length relationship will not be affected by the operational stress level; thus, the K_{cr} = 30 ksi √in curve in Example 9.7.1 describes the relationship for this example.

The corresponding crack growth-life curves for characteristic stress levels ($\overline{\sigma}$) of 15, 20, and 25 ksi are presented. As anticipated, the highest stress produces the fastest crack growth-life behavior. Based on Equation 9.7.4, the curves are related to each other by a life factor given by

$$\frac{L_2}{L_1} = \left(\frac{\overline{\sigma}_1}{\overline{\sigma}_2}\right)^n$$

where the lives L_1 and L_2 are calculated at the same crack length (any choice of a_{cr} applies) for characteristic stress levels $\overline{\sigma}_1$ and $\overline{\sigma}_2$.

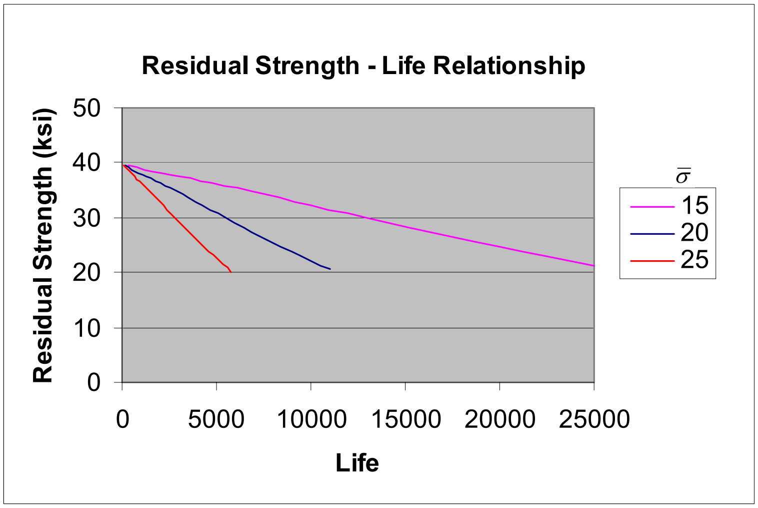


Effect of Stress Level on Crack Growth-Life Relationship

If one cross correlates the crack length-life behavior with the K_{cr} = 30 ksi √in residual strength-crack length behavior, then the residual strength-life behavior is as presented below. Note that the residual strength capability decays more slowly for the lower characteristic stress levels. As a method for predicting the residual strength-life behavior as a function of stress level, one could utilize the baseline curve identified for $\overline{\sigma}$ = 20 ksi and Equation 9.7.9 to provide the appropriate life factor (at any given residual strength level). For example, the $\overline{\sigma}$ = 15 ksi residual strength-life curve is displaced by a factor of

$$\frac{L_{15}}{L_{20}} = \left(\frac{20}{15}\right)^{3.0} = 2.37$$

from the $\overline{\sigma}$ = 20 ksi residual strength-life curve (check this at σ_{res} = 30 ksi where $L_{20} \cong 5400$ and where $L_{15} \cong 12800$.) Thus, one could construct residual strength-life curves as a function of characteristic stress levels by generating a baseline curve and applying the life factor.



Effect of Stress Level on Residual Strength-Life Relationship

EXAMPLE 9.7.3 Effect of Pre-Exponential Constants

This example and Example 9.7.4 collectively consider the effect of modifying the material's crack growth rate response on the residual strength capability. In both examples, the baseline conditions stated in Example 9.7.1 are used unless otherwise specified.

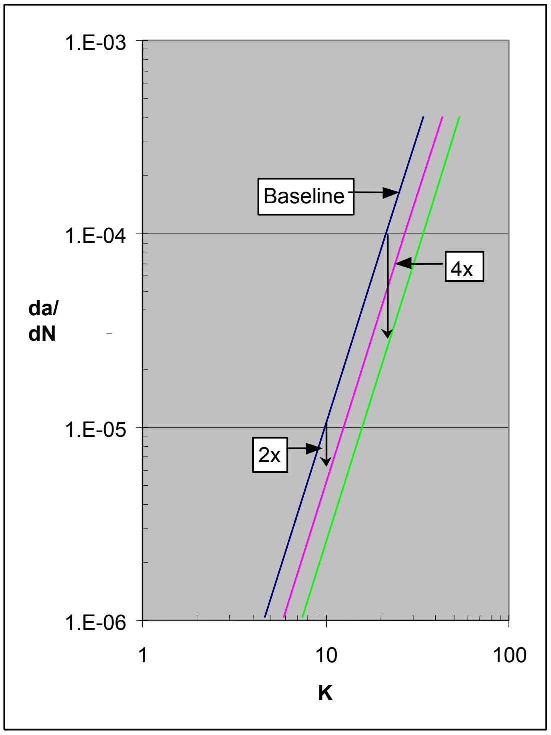
It is noted that the crack growth rate resistance can be changed independent of the fracture toughness (fracture resistance), so that the residual strength-crack length relationship is again given by the K_{cr} = 30 ksi √in curve in Example 9.7.1.

In a somewhat decoupling fashion, the effect of varying the coefficients in the growth rate equation, (Equation 9.7.2)

$$\frac{da}{dN} = C\overline{K}^{n}$$

are considered separately. In this example, only the constant C is varied to reflect decreasing the crack growth rate response in a systematic manner from the baseline condition where $C = 1 \times 10^{-8}$. In Example 9.7.4, the effect of varying the exponent n is considered.

A change in the constant C is equivalent to shifting the crack growth rate curve to a new position but with the same slope. If the constant C is reduced by a factor of 2, i.e. C=0.5×10^{-8}, then the growth rate da/dN decreases by a factor of 2.



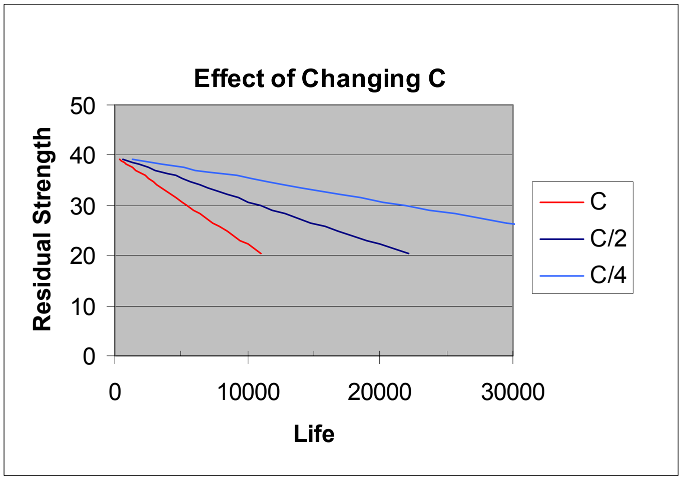
Variation of Crack Growth Behavior Resulting from
a Shift in the Power Law Curve

Based on an analysis of Equation 9.7.4, it is seen that the life difference that results from a change in C can be expressed as a life ratio

$$\frac{L_2}{L_1} = \frac{C_1}{C_2}$$

Thus, if a baseline crack growth-life curve and a baseline residual strength-life curve exist, new curves can be generated by factoring the lives from the baseline condition to the new material conditions using this equation.

The figure below describes the residual strength-life curves for the baseline and two lower values of the constant *C*. From the figure, it is seen that the increased crack growth resistance, i.e. lower *C* values, results in slower rates of residual strength decay. The new curves are exactly a factor of two and of four removed from the baseline curve.

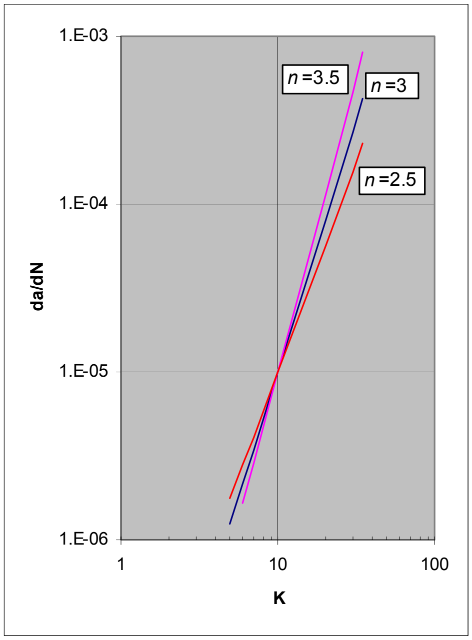


Effect of Constant *C* on Residual Strength-Life Relationship

Increasing the material's crack growth resistance has an immediate effect of increasing the number of flights (amount of flight hours) until the residual strength capability decays to the residual strength requirement.

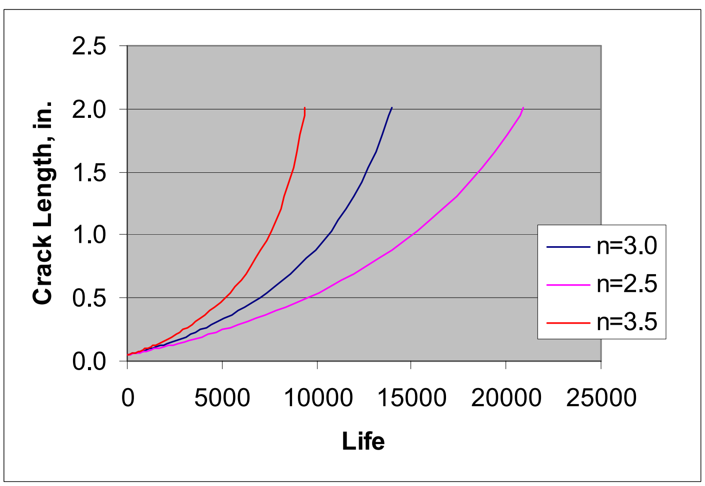
EXAMPLE 9.7.4 Effect of Exponential Constant

In this example, the exponential constant (n) in Equation 9.7.2 is varied along with the constant C to reflect a defined rate of crack growth (da/dN = 10×10^{-6} in/cyclic unit) for a given characteristic stress-intensity factor level of $\overline{K}$ = 10 ksi √in. The baseline constants of $n = 3.0$ and $C = 1 \times 10^{-8}$ yield an equation which passes through the point ($10, 10 \times 10^{-6}$). This figure illustrates the three choices of n considered in this example.

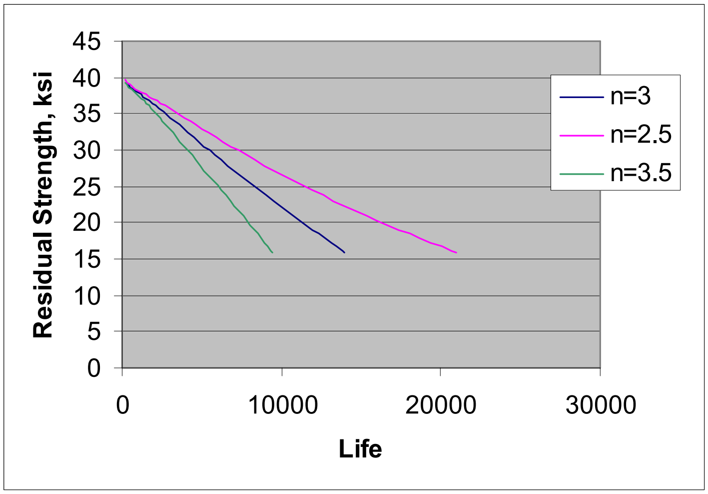


Crack Growth Curves Shown Passing Through Common Point

For this baseline geometry, fracture toughness level, and stress level, the characteristic stress-intensity factor varies between about 15 and 30 ksi √in as a result of the crack growth change. When the common point for the power law equations is located at a stress-intensity factor level that corresponds to a crack length within the crack length interval associated with the life calculation, one can not immediately interpret the effect of the crack growth rate behavior. However, based on the crack growth rate behavior defined in the figure, the curve with $n = 3.5$ will yield crack growth rates faster than the baseline throughout the crack length interval of interest. Thus, for the conditions stated, an engineer would expect a more accelerated crack growth behavior and a more rapidly decaying residual strength behavior for the $n = 3.5$ material than for the baseline. The following figures bear out this expectation.



Effect of Exponent n on the Crack Growth Life Relationship



Effect of Exponent n on the Residual Strength-Life Relationship

One observation made in studying the residual strength-life behavior presented in the figure is that the decay in residual strength is slightly nonlinear in the long life region for the two non-baseline crack growth rate behaviors. For the $n = 2.5$ material, the residual strength-life curve is slightly concave up while the $n = 3.5$ material produces a slightly concave down shape. Thus, a second factor that produces nonlinear decay effects is the exponent n. Generally speaking, nonlinear decay effects would be expected when the crack growth rate behavior can not be described by a power law equation with $n = 3$. While the nonlinear behavior is evident, it is important to note that it is slight. As a result, local regions of the residual strength life curve can be easily described by linear line segments, and the procedures presented in Examples 9.7.1, 9.7.2, and 9.7.3 can be utilized to extrapolate from a segmented baseline curve.

The rate of decay in residual strength as a function of service loading has been shown by the above examples to be an important function of material behavior and of load level. The residual strength decay rate can also be significantly affected by geometric parameters and loading conditions. In Example 9.7.5, the effect of global and crack geometry is considered; and then in Example 9.7.6, the effect of localized fastener loading is evaluated.

EXAMPLE 9.7.5 Effect of Geometrical Parameters

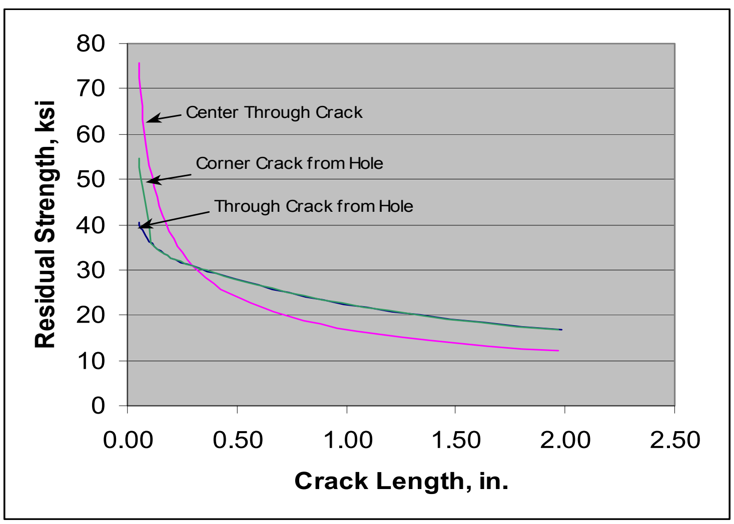
Using the through-the-thickness, radially-cracked, open hole geometry (shown in Example 9.7.1) as the baseline geometry, two other geometrical configurations are considered: (1) the through-the-thickness, center-crack and (2) an open hole with a radial crack which transitions from a one-quarter-circular, corner-crack shape to a through-the-thickness-crack shape. In all cases, the width of the structure is considered sufficient so that it does not influence the results. Baseline material properties (K_{cr}, C, and n), initial crack length (a_o), and characteristic stress level ($\overline{\sigma}$) are as defined in Example 9.7.1 and apply to all three geometries. The center crack geometry does not have a central (starter) hole; its total initial length is $2a_o$. The radius (r) of the hole with the transitioning crack is 0.125 inch, the same as the baseline geometry.

The information presented at the introduction of this section described how the residual stress relationships could be developed using Equations 9.7.1 and 9.7.4 and the stress-intensity factor coefficient for the geometry. The only factor that changes as a function of geometrical parameters is the stress-intensity factor coefficient; Example 9.7.1 provides this coefficient for the baseline case. For the through-the-thickness, center crack configuration in an infinite plate, the stress-intensity factor coefficient is given by

$$\frac{K}{\sigma} = \sqrt{\pi a}$$

The case of the transitioning corner crack requires that the crack growth shape be known throughout the interval of crack growth. The stress intensity factor solution for this geometry is given in Section 11.

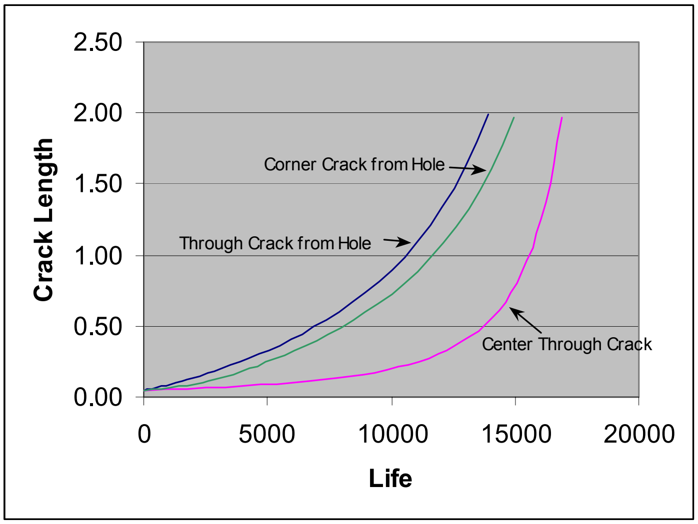
When the stress-intensity factor coefficients for the given geometries are utilized in conjunction with Equation 9.7.1, the residual strength-crack length relationships are determined (K_{cr}=30 ksi √in). As expected, the transitioning corner crack geometry exhibits residual strength that is greater than that of the through-the-thickness crack geometry (baseline) for shorter cracks. For crack lengths greater than 0.250 inch, the transitioning radial crack and baseline configurations exhibit the same residual strength (since the stress-intensity factor coefficients are the same here). One interesting feature of this plot is that the residual strength of the center crack configuration is higher than the radially cracked holes for short crack lengths but rapidly decreases with crack length and eventually falls below the residual strength exhibited by the cracked hole. One might puzzle through this observation by noting that the center crack has a total length of 2*a*, whereas the radially cracked hole has an equivalent length of (*a*+2*r*).



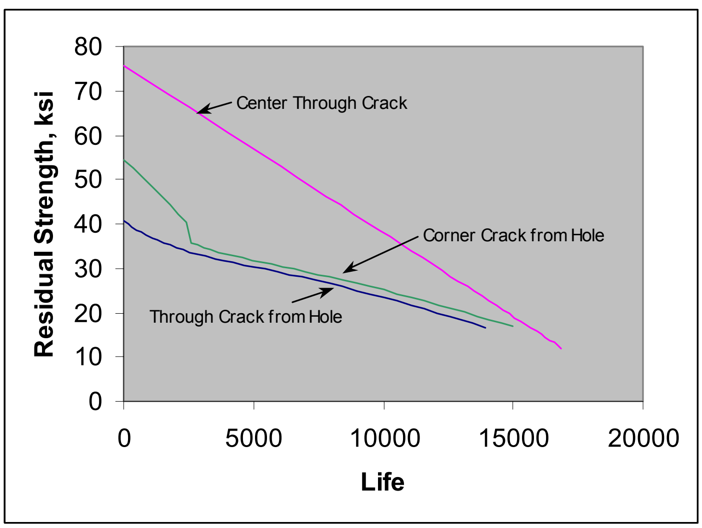
Effect of Geometry on the Residual Strength-Crack Length Relationship

Equation 9.7.4 was utilized to calculate the crack growth life relationships for the three geometries and these are shown below. Because the stress-intensity factor for the through-the-thickness radial crack is initially higher than those of the other two configurations, the baseline configuration exhibits the fastest crack growth behavior. The transitioning radial-corner-crack configuration initially exhibits slower crack growth behavior than the baseline but eventually these two crack growth curves become parallel (when the stress-intensity factor is the same, i.e. when $a > 0.250$ inch). The center crack configuration exhibits the slowest initial growth behavior, and this is primarily because the stress-intensity factor for small crack lengths is substantially below that of the other two configurations.

By cross-correlating the information presented in these figures, one is able to construct the residual strength-life relationships shown in the next figure. As anticipated, the baseline configuration has the lowest residual strength capability and the center crack configuration exhibits the highest residual strength capacity. Both the baseline and center crack configurations are also shown to exhibit an extensive region of linear residual strength decay as a function of time-in-service. The nonlinear residual strength decay exhibited by the transitioning radial corner crack is attributed to the gradient in the stress-intensity factor coefficient for relatively short cracks and the transition to a through crack. Based on observations in this and other examples in Section 9.7, it would appear that one of the most important factors contributing to nonlinear behavior is the severity of stress-intensity factor gradient (as a function of crack length).



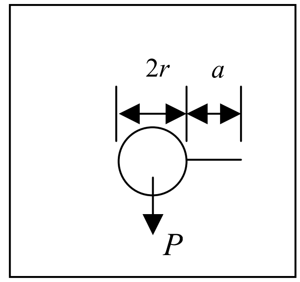
Effect of Geometry on the Crack Growth-Life Relationship



Effect of Geometry on the Residual Strength-Life Relationship

EXAMPLE 9.7.6 Effect of Hole Loading

As a means of evaluating the effect of fastener loading on the residual strength, this example combines the baseline remote loading configuration described in Example 9.7.1 with localized pin loading shown here. To calculate residual strength, the baseline material properties are utilized in Equations 9.7.1 and 9.7.4 along with the stress-intensity factor associated with the combined loading.



$a_0 = 0.050$ inch
$r = 0.125$ inch
$K_{IC} = 30$ ksi $\sqrt{\text{in}}$
$\frac{da}{dN} = 1 \times 10^{-8} \overline{K}^{3.0}$

Because the structural response is linear elastic, stress-intensity factor solutions for the remote and localized loadings can be added to obtain the stress-intensity factor for the combined loading; thus,

$$K_{Total} = K_{remote} + K_{local}$$

where K_{remote} is obtained from the product of the remote stress ($\overline{\sigma}$) and the stress-intensity factor coefficient for a wide plate given in Example 9.7.1, so that:

$$K_{remote} = \overline{\sigma}\beta_{remote}\sqrt{\pi a}$$

and $$\beta_{remote} = \left(0.7071 + 0.7548y + 0.3415y^2 + 0.642y^3 + 0.9196y^4\right)$$

where $$y = \left(\frac{1}{1 + a/r}\right)$$

K_{local} is the stress-intensity factor associated with the pin loading. From Section 11, this stress intensity factor is given by

$$K_{loacl} = \frac{P}{2rt}\beta_{PR}\sqrt{\pi a}$$

where

$$\beta_{PR} = \beta_{remote}\left(\frac{r}{W}\right) + G_1$$

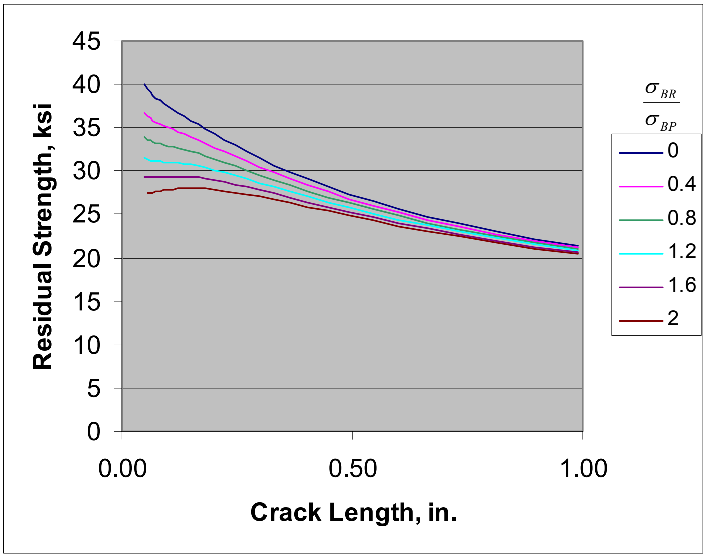
and

$$G_1 = \left(0.078y + 0.7588y^2 - 0.4293y^3 + 0.0644y^4 + 0.651y^5\right)$$

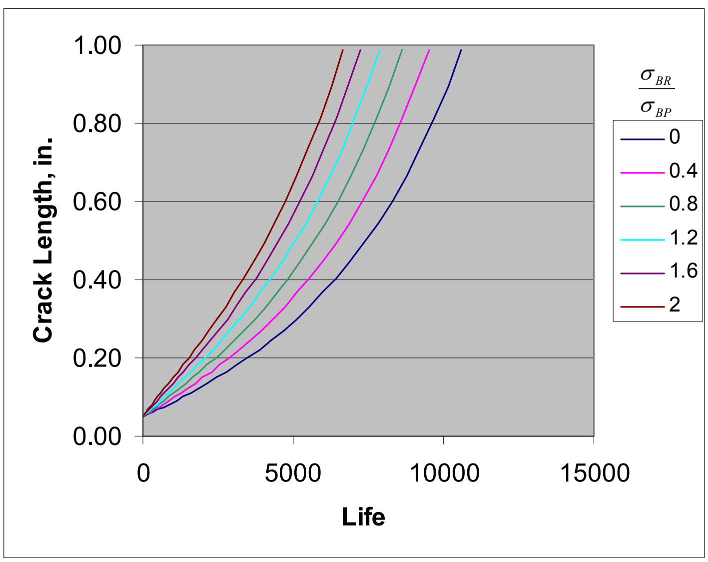
As a method of comparing the effect of pin loading in conjunction with remote stress loading, the bearing to bypass ratio was used. The bearing to bypass ratio is the ratio between the bearing stress ($P/2rt$) and the remote stress $\overline{\sigma}$, i.e.

$$\frac{\sigma_{BR}}{\sigma_{BP}} = \frac{\frac{P}{2rt}}{\overline{\sigma}}$$

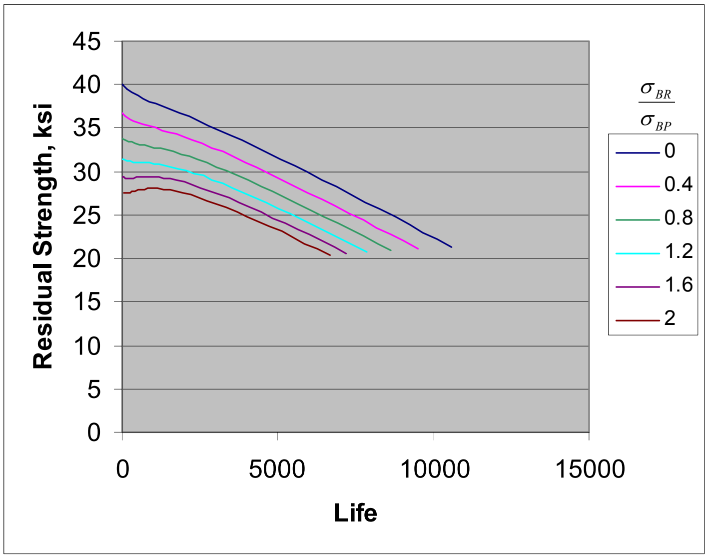
Various combinations of this ratio were chosen and the residual strength relationships were then generated using Equations 9.7.1 and 9.7.4. The residual strength-crack length relationships are shown for K_{cr}=30 ksi $\sqrt{\text{in}}$; and, the crack length-life are shown for $C=1\times10^{-8}$ and n=3.0. By cross-correlating this information, one can generate the residual strength-life relationships, as shown.



Effect of Pin Loading on the Residual Strength-Crack Growth Relationship



Effect of Pin Loading on the Crack Growth-Life Relationship



Effect of Pin Loading on the Residual Strength-Life Relationship

Based on the results presented in these figures, it would appear that bearing to bypass ratios less than 0.4 cause a relatively small change in the residual strength/crack length/life relationships. As the bearing to bypass ratio increases from 0 to 2, (a) the residual strength decays very rapidly in the short crack region, (b) a significant reduction occurs in the crack growth-life curves, and (c) the residual strength-life curves are progressively lower. The collective sum of these observations indicate that when substantial hole loading is present, it is necessary to account for the hole loading when assessing the residual strength and crack growth life behavior.